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Astrophysics for a Ten-Year-Old Mind

Michael Chabin

Try an experiment. Over the next couple of days, ask a random selection of the people you meet to tell you the difference between 1/4 and 1/5. Be sure to ask politely. Many of us have painful associations with random arithmetic problems given out of context. But if you ask nicely and find a broad range of people, you may notice, as we did, that the vast majority of those surveyed don't know the answer.

Radical members of the "Back to Basics" movement in the U.S. might conclude from this experiment that America's schools are failing and what is needed is a return to the kind of mathematics instruction that has made state lotteries so popular. But they would be wrong. Our survey says nothing about the schools. Almost every American fifth-grader can tell you the difference between 1/4 and 1/5.

What the survey does suggest is that most adults never subtract fractions in that form and they have quite understandably forgotten the method.

Now try a different experiment. Show your subjects the equation e = mc2 and ask what it means. Prepare yourself for a pleasant experience: Not only do people know a lot about this arcane piece of physics, but they like talking about it.

The equation is the most famous in science. It can be found in magazines, comic strips, textbooks, graffiti, posters, and science fiction novels. Remember the little bespectacled chick often under the care of the cartoon rooster Foghorn Leghorn? He had a t-shirt proclaiming this seemingly arcane physics. An astonishing number of people recognize it. Most associate it with Albert Einstein and many will say it has to do with mass and energy. A heartening number of students and teachers will even tell you the equation states that energy is equal to mass times the speed of light squared.

All of which might lead radical members of the education establishment to claim that American science education is not as bad as recent international surveys indicate and that in order to continue such remarkable successes teachers should be given raises and more professional development time. But they would be just as wrong as the "back to basics" advocates. What people know about Einstein's equation, like much of the useful and practical science they know, is not part of the curriculum and was not learned in school.

In fact, though this is by far the most recognized equation in history, it is not recognized as an equation at all. Rather e=mc2 has become a trademark of science in much the same way golden arches are a trademark of McDonalds. One way to see this is to rearrange it to e= c2m and ask people to tell you what that is. Another way is to ask them to use it.

The equation is almost never shown as:

Energy (in joules) = Mass (in kilograms) X The Speed of Light Squared (in meters squared per second squared)

Maybe that is why many of those who proudly tell you "energy is equal to mass times the speed of light squared" will blush if you then say, "Okay, then, so how much energy will one liter of water yield if it is converted entirely into energy?" Clearly, knowing the equation is one thing, using it is for specialists. Accepting this contradiction is not their fault. Much science is taught by memorization.

Still, it is a shame students don't have a chance to use the equation. It is elegant. It is powerful. It lets them answer exotic questions. And, the "back to basics" movements notwithstanding, learning to use it exploits just the kind of mathematics that can be expected of early middle schoolers.

In 1989 the National Council of Teachers of Mathematics (NCTM) released a set of standards which, outside of California, are used to guide curriculum development. Of the standards for grades 5-8, Standard 9 includes understanding the concepts of "variable, expression, and equation." Scientific notation is implicitly part of the standards and is used in examples given, as is the metric system.

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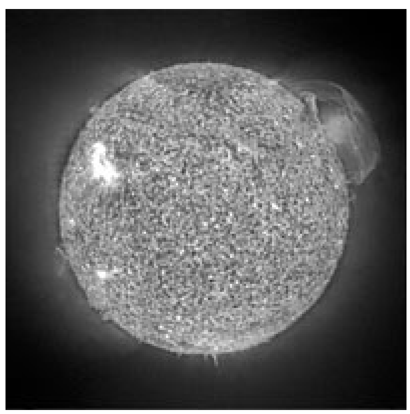
There is no math in Einstein's equation that goes beyond this, so why isn't it taught to ten-year olds?

Perhaps surprisingly to people outside the field, many educators object to "teaching relativity in grade school" on the grounds that students as young as ten lack the intellectual sophistication necessary to form a meaningful notion of the equivalence between mass and energy. The argument is that children learn science more readily if it pertains to the visible world.

Adherents to this view are too numerous, too respectable, and too well funded to discount. But if you spend much time around ten-year olds, you' ll find many of them already know about atoms, black holes, and antimatter. Their ideas may be a bit vague in detail, just as their ideas of life in London or of the relative sizes of Spain and Brazil are still forming, but children are exposed to media other than textbooks, and through their extracurricular sources, they have acquired some sophisticated interests.

Of course, you don't have to rely on either argument. If you have access to a ten-year old, you can do the experiment yourself.



$E=mc^2$ on a big scale. Inside the Sun, an enormous amount of matter is converted to energy every second. Indeed, the Sun's energy output is equivalent to about four trillion trillion 100 watt light bulbs shining at the same time. Image courtesy of SoHO/EIT consortium. SoHO is a project of international cooperation between ESA and NASA.

Begin by telling her that one of the remarkable results of the century is that matter and energy are two forms of the same thing. Neither can be destroyed but, under special circumstances, either can be turned into the other. This is important. All the matter in the universe condensed out of pure energy over a period of about 700,000 years shortly after the Big Bang, and today, stars shine because matter at their centers is being slowly converted back into energy. Both annihilation of matter into energy and condensation of matter out of energy have been demonstrated in the laboratory.

Explain that one of the most amazing things about all this is that a simple equation will tell you exactly how much energy a given amount of mass contains. Invite your partner to write e=mc2 at the top of a piece of paper. Then take a moment or two for her to tell what she knows about the equation.

Then ask her to write the following just below the first equation.

Energy (in joules) = Mass (in kilograms) X The Speed of Light Squared (in meters squared per second squared)

Be sure she knows what a meter is and then explain that you want to figure out how much energy is contained in a one-liter bottle of water if the entire mass is converted to energy. Begin with the speed of light.

Light travels three hundred million meters in a second so now she can write:

energy = mass X $(300{,}000{,}000)^2$, or $e = m\ (3 \times 10^8)^2$

You can leave the units for now, but we cannot forget them at the end. The next step, of course, is to square the speed of light. That produces an even bigger number and makes the need for scientific notation clear. Once that is done, however, the result is a constant that simplifies the equation to:

e = m (90,000,000,000,000,000), or $e = m\ (9 \times 10^{16})$

Mass is next. It is given in kilograms, and, since a liter of water is defined to have a mass of one kilogram, most kids have no trouble with this unit. Because you want to know how much energy is in one liter of water, the mass term is equal to 1 and the equation becomes:

$e = 1\ (9 \times 10^{16})$, or $e = 9 \times 10^{16}$ joules

That's it. That's the answer. And that is all there is to it. Square the speed of light in meters per second, and you get the energy provided by a kilogram of mass. Swapping values for mass will let kids calculate the energy equivalence for everything from the Sun to a hot dog.

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A joule expended continuously for one second is a watt, which means a liter of water, converted to energy could power a 1 watt light bulb for 9 • 1016 seconds, or 2.5 • 1013 hours. Of course if it were a 100 watt bulb it would only burn for 2.5 x 1011 hours, and you might find it more interesting to burn 1011 light bulbs for two and a half hours instead. It is then reasonable to ask your partner how large an area that many light bulbs would cover. She'll find, after a little work (especially if she hasn't seen square roots yet), that if she allows an area of about 10 cm by 10 cm for each bulb, she can tile a square roughly thirty-one kilometers on a side with 1011 light bulbs.

What you've just done is arithmetic. Now for the science.

What would happen if you turned all those light bulbs on for two and a half hours?

Would it explode? The amount of energy released would be a little less than a tenth of the energy released in the enormous hydrogen bomb tests of the early 1960s, but the area over which you are releasing it is larger, and you are releasing it over a longer time.

Or would it generate tornadoes? You are releasing less than one one-hundredth of the energy in a typical hurricane, but hurricanes form over vast stretches of the ocean and typically take a week or so to blow themselves out. You are releasing the energy over a comparatively tiny area in less than three hours.

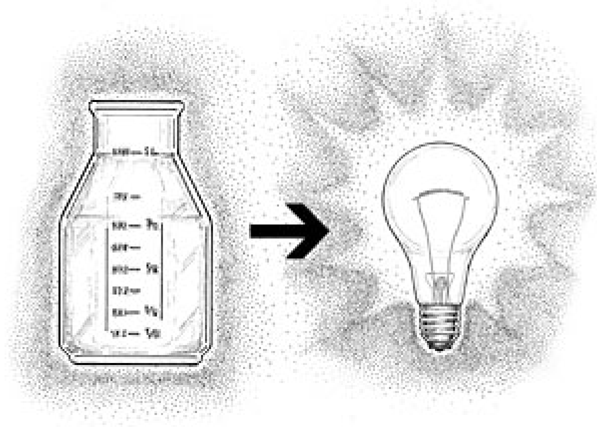
Or would most of the energy escape through the clear sky as a brilliant beacon? Wondering about such possibilities are the roots of science and very appropriate for ten-year olds and curious adults.

Incidentally, the average ten-year old uses about the same amount of energy as a 100 watt bulb, which is why classrooms can warm up so readily. In fact, there is a direct equivalence to food calories which makes a broad range of interesting and silly calculations possible. One joule is equal to about four food calories.

Of course, the calculations can also be serious and scientific. A little bit of algebra that is very appropriate to introduce to fifth graders will allow you to find the mass you need to meet any particular energy requirement. The equation now looks like m = e/c2. For example, given that the Sun emits $4 \text{ X } 10^{26}$ joules per second, how much mass does it convert each second in order to burn? If the Sun is $5 \text{ X } 10^{9}$ years (or $1.6 \text{ X } 10^{17}$ seconds) old, how much mass has it used so far?

It doesn't matter very much what the calculations are. It doesn't matter if the distances or quantities are accurate. What does matter is that students will be exercising the math appropriate to their grade level on one of the most remarkable equations known. Without a refresher from time to time it is unlikely that they will remember the details into adulthood. They don't need to. What is important is that having played with the equation once, they will know that it is tractable, accessible, and so much more than a trademark of science.

MICHAEL CHABIN develops web-based animations that teach math to kids. Some of his animations can be seen at http://www.hypatia.org. He was once operator of the 2.3 meter telescope at Steward Observatory for the University of Arizona, and he is married to an astronomer. His email address is chabin@hypatia.org.



Minding Your Joules

The problem is, of course, understanding what your answer means. Energy is given in joules and not many kids know what a joule is. But they should; it is an extremely intuitive unit. A joule is the effort required to push one kilogram over a meter at an acceleration of one meter per second squared. To get a feel for it, pick up a liter bottle of water and toss it ten centimeters into the air. You've just expended about a joule. Here are some others:

on (Very) Tiny scales...

- breaking a single bond in human DNA requires just 10-20 joules
- a firing neuron needs only 10-9 joules
- for a hop, a flea requires about 10-7 joules

on human scales...

- shooting a basketball jump shot from the free-throw line takes about 15 joules
- throwing a fastball requires 120 joules
- rolling a strike with a bowling ball will cost you 230 joules

on planet scales...

- the largest hydrogen bomb ever tested produced 2.4×10^{17} joules
- a typical hurricane amounts to about 3.8×10^{19} joules

on astronomical scales...

- a supernova releases around 10^{44} joules
- one estimate gives the energy yield of the Big Bang as 10^{68} joules